

Conference in Hyperbolic Geometry and Geometric Analysis
October 15-17, Wesleyan University

Titles and Abstracts

- **Ian Agol**

Title: “Rank and Heegaard genus of arithmetic 3-manifolds.”

Abstract: We show that there are finitely many arithmetic 3-manifolds of Heegaard genus $\leq g$, up to taking (subdihedral) cyclic covers of (semi)fibered arithmetic 3-manifolds (where the fiber has genus $(g-1)/2$). Assuming the Salem number conjecture (or a weakened version which states that there is a minimal Salem number), we show that there are only finitely many arithmetic 2-generator 3-manifolds.

- **Jeff Brock**

Title: “Pants decompositions of surfaces and volumes of hyperbolic 3-manifolds: a postmodern perspective.”

Abstract: The close comparisons between the convex core volumes of hyperbolic 3-manifolds and distances in the complex of pants decompositions of a surface that arise in the quasi-Fuchsian and fibered cases suggest possible generalizations to the compressible boundary case and to the case of closed hyperbolic 3-manifolds with certain kinds of Heegaard splittings. In this talk I will discuss new techniques to treat the original cases that are better adapted to this more general setting. This is joint work with Juan Souto.

- **Ken Bromberg**

Title: “The density conjecture for Kleinian groups with bounded geometry.”

Abstract: The density conjecture states that every finitely generated Kleinian group is approximated by geometrically finite groups. We will discuss an approach to this conjecture that does not depend on the Ending Lamination Conjecture. This is joint work with Juan Souto.

- **David Dumas**

Title: “Grafting, Pruning, and the Teichmüller Geodesic Involution.”

Abstract: Grafting is a cut-and-paste operation on hyperbolic surfaces.

In this talk we present results about the large-scale behavior of ”pruning” (the inverse of grafting) which have application to the theory of complex projective structures on Riemann surfaces. In particular we show that pruning extends continuously to a map of the Thurston boundary of Teichmüller space, and we identify the boundary values with the Teichmüller geodesic involution.

- **David Gabai**

Title: “Shrinkwrapping and the taming of hyperbolic 3-manifolds.”

Abstract: Joint work with Danny Calegari. We discuss, shrinkwrapping a new technique for finding CAT(-1) surfaces in hyperbolic 3-manifolds. We use it to show that a complete hyperbolic 3-manifold with finitely generated fundamental group is geometrically and topologically tame.

- **John Holt**

Title: “Non-accumulation of components of deformation spaces of hyperbolic 3-manifolds.”

Abstract: By work of Ahlfors-Bers, Marden, Sullivan, Maskit, Kra and Thurston, the components of the interior of the space $AH(M)$ of isometry classes of marked hyperbolic 3-manifolds homotopy equivalent to a compact hyperbolizable 3-manifold M are indexed by the set of marked oriented homeomorphism types of compact hyperbolizable 3-manifolds homotopy equivalent to M . Canary and McCullough classified for which manifolds this set is infinite - in particular, if M has compressible boundary (and is not one of several simple exceptions) there are infinitely many components to the interior of $AH(M)$. Anderson-Canary-McCullough showed that when M has incompressible boundary the closures of the components of the interior of $AH(M)$ do not accumulate.

We prove that the closures of the components of the interior of $AH(M)$ do not accumulate, extending the result of Anderson-Canary-McCullough.

Joint with with Gero Kleineidam.

- **Zheng Huang**

Title: “On Asymptotic Weil-Petersson Geometry of Teichmüller Space.”

Abstract: We will study various properties of curvatures of the Weil-Petersson metric near the frontier space of Teichmüller space. Main results include estimates on sections with asymptotic negative infinite curvature and asymptotically flat sections.

- **Linda Keen**

Title: “Hyperelliptic Handlebodies.”

Abstract: This talk is based on joint work with Jane Gilman. A Kleinian group naturally stabilizes certain subdomains and closed subsets of the closure of hyperbolic three space and yields a number of different quotient surfaces and manifolds. Some of these quotients have conformal structures and others hyperbolic structures. For two generator free Fuchsian groups, the quotient three manifold is a genus two solid handlebody and its boundary is a hyperelliptic Riemann surface. The convex core is also a hyperelliptic Riemann surface. We find the Weierstrass points of both of these surfaces. We then generalize the notion of a hyperelliptic Riemann surface to a “hyperelliptic” three manifold. We show that the handlebody has a unique order two isometry fixing six unique geodesic line segments, which we call the *Weierstrass lines* of the handlebody. The Weierstrass lines are, of course, the analogue of the Weierstrass points on the boundary surface. Further, we show that the manifold is foliated by surfaces equidistant from the convex core, each fixed by the isometry of order two. The restriction of this involution to the equidistant surface fixes six *generalized Weierstrass points* on the surface. In addition, on each of these equidistant surfaces we find an orientation reversing involution that fixes curves through the *generalized Weierstrass points*.

- **Chris Leininger**

Title: “A combination theorem for Veech subgroups of the mapping class group.”

Abstract: Veech groups are “Fuchsian” subgroups of the mapping class group defined in terms of Teichmüller geometry. I’ll discuss a combination theorem for Veech groups analogous to the first Maskit combination theorem for Kleinian groups in which the amalgamating subgroup is of parabolic type. As a corollary we obtain subgroups of the mapping class group (for all genus at least 2) isomorphic to non-abelian surface groups, in which all but one conjugacy class of elements (up to powers) is pseudo-Anosov. This is joint work with Alan W. Reid.

- **Gaven Martin**

Title: “The Generalised Lichnerowicz Conjecture: Rational maps of manifolds.”

Abstract: The Lichnerowicz conjecture, solved affirmatively by Lelong-Ferrand in the 70’s asked if the n -sphere was the only closed manifold with non-compact conformal automorphism group. The generalised problem replaces conformal (i.e. 1-1) by rational. It was widely believed that rigidity theory implies that no manifolds (other than the n -torus) in dimension $n > 2$ admitted non-injective maps which were rational with respect to some measurable conformal structure. However examples were constructed on the n -sphere, leading to an interesting higher dimensional analogue of the Fatou-Julia theory of iteration of rational endomorphisms of the Riemann sphere. Here we discuss some of the basic theory and present our most recent results toward the solution of this problem (including the complete solution in 3-dimensions). This is joint work with V. Mayer (Lille), A. Hinkannen (Illinois) and K. Peltonen (Helsinki).

- **Bernie Maskit**

Title: “On Neoclassical Schottky Groups” (joint work with Rubén Hidalgo).

Abstract: From the point of view of 3-manifolds, a Schottky group of rank g is a handlebody of genus g endowed with a geometrically finite complete hyperbolic metric with injectivity radius bounded from below. More precisely, the Schottky group is the universal covering group of the handlebody. The handlebody, or the Schottky group, is classical if it contains g totally geodesic disjoint homologically independent properly embedded discs. More precisely, topologically, the discs are closed, disjoint and properly embedded in the compact handlebody; geometrically, the interiors of the discs are totally geodesic at every point.

A noded Schottky group of rank g is the universal covering group of a handlebody of genus g endowed with a geometrically finite complete hyperbolic metric. The handlebody, or the noded Schottky group, is neoclassical if it contains g totally geodesic disjoint, except perhaps at the boundary, properly embedded homologically independent discs. More precisely, the discs are compact and properly embedded in the compact handlebody; they may touch without crossing at a finite number of points on the boundary; as above, the interiors of the discs are totally geodesic at every point.

It was shown by Marden that there exist non-classical Schottky groups; explicit examples were given by Yamamoto. We start with an explicit example of a non-neoclassical noded Schottky group. We show that there are infinitely many non-classical noded Schottky groups that can be analogously constructed, and we show that, for all of these, all “nearby” Schottky groups are necessarily non-classical.

- **Howie Masur**

Title: “Rational Billiards, and the $SL(2, \mathbb{R})$ action on the moduli spaces of Abelian differentials.”

Abstract: By a well-known construction, billiards in a polygon with vertex angles a rational multiple of π gives rise to a flat surface with cone angle singularities (Abelian differential) and the billiard flow gives rise to a flow by straight lines on the flat surface. The group $SL(2, \mathbb{R})$ acts on the moduli space of Abelian differentials. Most of the recent work on the subject of the dynamics of billiards and flat surfaces arises by studying this action. I will attempt to survey recent results in the subject.

- **Yair Minsky**

Title: “Geometry of Weil-Petersson geodesics”

Abstract: To a Weil-Petersson geodesic ray in Teichmüller space one can associate an “ending lamination”, in rough analogy with the vertical foliations of Teichmüller geodesic rays, and with ending laminations of hyperbolic 3-manifolds. One may conjecture that this lamination determines the ray up to asymptotic class; so far this is only known in special cases. We will provide more questions than answers about the structure of these rays. Joint work with Jeff Brock and Howard Masur.

- **Dragomir Saric**

Title: “Extremal Maps Between Hyperbolic Laminations.”

Abstract: A Riemann Surface Lamination is called hyperbolic if it supports a metric with constant curvature -1 which is compatible with the complex structure. Consider a homotopy class of quasiconformal maps between two hyperbolic laminations. We investigate under which conditions there exists a quasiconformal map with the smallest dilatation in the homotopy class. Such map is called extremal.

Given a holomorphic quadratic differential on a laminations and a positive number less than 1, one can define a Beltrami coefficient on the laminations. Teichmüller showed that such Beltrami coefficient on a compact Riemann surface determines uniquely extremal quasiconformal map of the surface onto another compact Riemann surface. This was extended for arbitrary Riemann surfaces by Reich and Strebel using a form of the length-area argument.

We extend the Reich-Strebel inequality to hyperbolic laminations with the transverse invariant measures. Using this inequality, it is standard to show that the above Beltrami coefficient produces a uniquely extremal quasiconformal map. We also discuss some additional consequences of the inequality. This work is a generalization of our previous work on the hyperbolic solenoid.